Response to Reviewers: A multi-physics Eulerian framework for long-term contrail evolution (MultiCon): Single plume analysis

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We sincerely thank the reviewers for dedicating their valuable time to reviewing our manuscript and for their careful and insightful comments, which we believe were very helpful in enhancing the quality of the paper.

For ease of reference, we have adopted the following format throughout our responses:

- Reviewer's comments
- Authors' responses

October 21, 2026

1 Reviewer 1

General comment. *"[. . .] I think most of the approximations are good, however I regret in the writing that little order of magnitude are given to justify them. It also lack of comparison with other model, there is one with CoCiP in one case, but there is results from APCEMM and CoCiP available in the literature as well as 2D LES results. It should have been great to have more comparison with other model. [. . .]"*

Authors' response. We thank the reviewer for the positive assessment of the framework and of the clarity of the derivation, and for the constructive suggestions. We address the two general points below; the specific remarks are answered point by point in the following section.

(i) Orders of magnitude. In the revised manuscript simplifying assumptions will be accompanied by the order of magnitude of the terms involved, and every test case will be better specified (temperature and lapse rate, pressure, depth and supersaturation of the ice-supersaturated layer, humidity outside it, shear, diffusivities, initial state, grid and time step, with units).

(ii) Comparison with other models. We agree that a single comparison with CoCiP was not sufficient. The revised manuscript contains a systematic comparison of MultiCon with CoCiP and APCEMM, the two models compared by Akhtar Martínez et al. (2025), which the reviewer refers to. CoCiP represents the widely used zero-dimensional Gaussian-plume approach, and APCEMM a two-dimensional model with sectional (polydisperse) microphysics that resolves the contrail cross-section and wind shear. To make the comparison meaningful we took care of the following points (Table R1):

- *Same initial state.* All models start from the same post-vortex state (ice number and mass per metre, plume dimensions), computed with the CoCiP wake-vortex parameterisation for the same aircraft and atmosphere.
- *Same atmosphere and diffusivities.* All models are driven by an identical, explicitly stated atmosphere through the pycontrails interface (CoCiP v0.64, APCEMM), and MultiCon and APCEMM use the same turbulent diffusivities.
- *Many conditions.* A baseline and 27 one-at-a-time perturbations cover the soot emission index, the supersaturation and depth of the ice-supersaturated layer, temperature and stratification, the humidity below the layer, the initial plume size, wind shear and diffusivities.

- *Common diagnostics.* Ice number and mass per metre, mean crystal radius, centre of mass, vertical and horizontal extent, and the extinction cross-section per metre (optical depth integrated across the contrail), together with centre-line number concentration and ice water content in the z–t plane.

Figures R1 shows the three models for one example (the baseline case). Figure R2 summarises all cases statistically. MultiCon and APCEMM, both of which resolve the vertical structure, produce the same plume structure and very similar crystal sizes and they respond consistently to the perturbations. APCEMM produces about twice the integrated extinction of MultiCon. CoCiP differs much more from both. The new Sect. 6 of the revised manuscript will also discuss the limitations of such comparisons, given the differences in model formulation and in the physical processes represented.

(iii) LES. We considered adding an LES comparison and decided against it in the present paper, for the following reasons.

1. *Cost and scope.* Resolving one contrail cross-section over several hours of the dispersion regime with LES requires orders of magnitude more computing time than the full set of simulations above. The present contribution is the derivation of a low-cost Eulerian framework, and a single LES realisation would add little statistical information compared with 28 harmonised cases with two established reference models.

2. *Reproducibility of published LES cases.* Published 2-D LES results (e.g. Unterstrasser and Gierens, 2010; Lewellen et al., 2014) depend strongly on the post-vortex initial state, the ambient profiles and the turbulence forcing, which are not available in enough detail to reproduce them with all three models. As shown in (ii), differences in the initial state alone can exceed differences between model formulations, so overlaying curves from the literature could be misleading.

3. *APCEMM as a resolved reference.* APCEMM solves the 2-D advection–diffusion problem with sectional microphysics and resolved shear. It is therefore the closest available reference for testing the separation ansatz and the monodisperse closure of MultiCon under identical conditions.

(iv) Scope of the paper. To avoid confusion about the scope, we would like to stress that the paper is primarily a theoretical contribution. It introduces an Eulerian framework that directly resolves PDE fields for large-scale contrail simulation at low computational cost, while allowing additional physical processes to be added as low-order PDE fields. Further contributions are a dynamic habit parameterisation obtained by correcting the ice-deposition formulas, which highlights the role of deposition kinetics in the ice-supersaturation budget, and a reduced-order turbulent bulk-settling velocity derived from multiphase-flow theory. The paper also includes habit-dependent particle settling, growth and sublimation, a prognostic ice-supersaturation field, and a characteristic-based offline treatment of wind shear (to be replaced by online shear feedback in future work). The new

comparisons place MultiCon in the context of established models and quantify where the present closures matter. They are not intended as a validation of an application-ready model; this version still lacks several processes (polydispersity, habit morphologies, ventilation, dynamic shear feedback). We state these limitations explicitly in the revised Sects. 6 and 7 to avoid over-interpretation of the comparison.

Table R1: Model comparison performed for the revised manuscript. All cases start from the same post-vortex state and use the same atmosphere; one parameter is varied at a time with respect to the baseline.

	values	rationale
Models		
MultiCon	present model (bulk settling, habit evolution), $\Delta z = 1.75\,\mathrm{m}$, $\Delta t = 10\,\mathrm{s}$	–
CoCiP	pycontrails v0.64, 0-D Gaussian plume, own turbulence closure	widely used reference
APCEMM	2-D sectional microphysics (38 size bins, log-normal initial distribution), resolved shear, via pycontrails	polydisperse, resolved reference
Baseline		
atmosphere	$T_0 = 222\,\mathrm{K}$, $\Gamma = 6.5\,\mathrm{K\,km^{-1}}$, $p_0 = 245\,\mathrm{hPa}$; ISSR 400 m deep with $s_\mathrm{env} = 0.10$; $s_\mathrm{env} = -0.15$ outside; no shear	
aircraft / initial state	B738-like, $\mathrm{EI}_n = 10^{14}\,\mathrm{kg^{-1}}$; $n_\mathrm{line} = 3.1\times10^{11}\,\mathrm{m^{-1}}$, $r_0 = 1.9\,\mu\mathrm{m}$, $\sigma_{y,0} = 9.5\,\mathrm{m}$, $\sigma_{z,0} = 29\,\mathrm{m}$	CoCiP wake-vortex parameterisation
One-at-a-time perturbations		
soot number emission index	10^{13}, $10^{15}\,\mathrm{kg^{-1}}$	initial crystal number
ISSR supersaturation	0.05, 0.20, 0.30	ambient forcing
ISSR depth	200, 800, 1600 m	fall-streak survival
temperature at reference level	212, 217 K	ambient forcing
humidity below/above ISSR	$s_\mathrm{env} = -0.05, -0.30$	sublimation of fall streak
stratification	$\Gamma = 0$ (isothermal), $3\,\mathrm{K\,km^{-1}}$	
initial plume size	$\sigma_{z,0} = 60$, 120 m; $\sigma_{y,0} = 50$, 150 m; both	post-vortex uncertainty
vertical wind shear	2, 4, $8\times10^{-3}\,\mathrm{s^{-1}}$	spreading
turbulent diffusivities	$D_{zz} = 0.05$, 0.3; $D_{yy} = 5$, $30\,\mathrm{m^2\,s^{-1}}$	closure uncertainty
initial state of the submitted paper	$n_\mathrm{line} = 3\times10^{10}\,\mathrm{m^{-1}}$, $a_0 = 1\,\mu\mathrm{m}$, $\sigma_{z,0} = 20$, $\sigma_{y,0} = 10\,\mathrm{m}$	link to previous results
CoCiP case of submitted paper	$T = 212\,\mathrm{K}$, RH_i=115 % in 400 m, shear $10^{-3}\,\mathrm{s^{-1}}$, 15 h	link to previous results
diagnostics	N, I, r_vol, z_cm, σ_z, σ_y, extinction per metre E, lifetime; $c_N(z,t)$, $\mathrm{IWC}(z,t)$	

Furthermore, in the revised version of the manuscript we will address the major technical com-

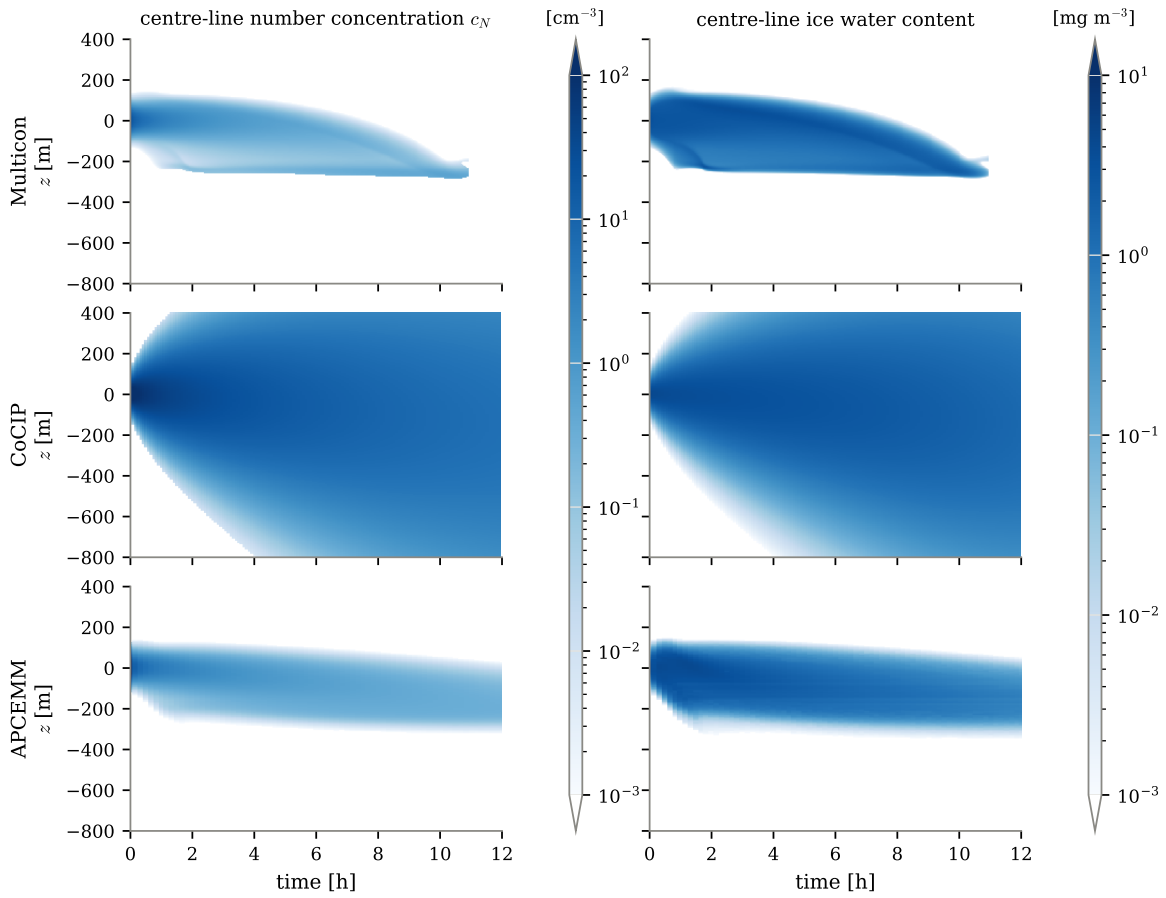


Figure R1: Example of the comparison (baseline case of Table R1): centre-line ice crystal number concentration c_N (left) and ice water content (right) in the height–time plane for MultiCon (top), CoCiP (middle) and APCEMM (bottom). Fields are shown where IWC $> 10^{-3}\,\mathrm{mg\,m^{-3}}$ and up to the end of the contrail lifetime. CoCiP fields are reconstructed from its Gaussian plume covariance; APCEMM fields are taken at the horizontal position of the plume centre. $z = 0$ is the plume centre at the end of the vortex phase; the ice-supersaturated layer spans $-200 < z \leq 200\,\mathrm{m}$.

ments by clarifying the assumptions and their justifications, providing relevant order-of-magnitude estimates, elaborating on the population balance formulation and its simplifications, and improving the descriptions of the numerical discretization, convergence assessment, and characteristic-based approach.

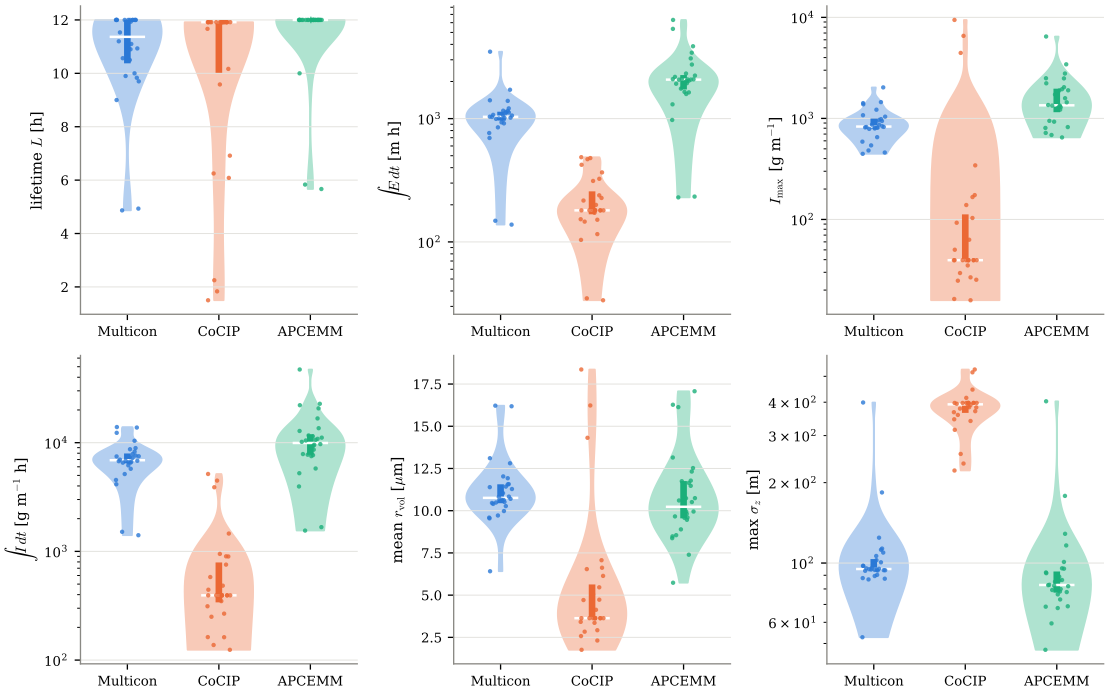


Figure R2: Statistical summary of the comparison over the 28 harmonised cases of Table R1 for MultiCon, CoCiP and APCEMM: contrail lifetime L (simulations end at 12 h), time-integrated extinction cross-section per metre $\int E\,dt$, maximum and time-integrated ice mass per metre ($I_{\max}$, $\int I\,dt$), ice-mass-weighted mean crystal radius and maximum vertical standard deviation of the plume. Shaded: kernel density; thick bar: interquartile range; white line: median; dots: individual cases.

2 Reviewer 2

General response: We thank the reviewer for taking the time to review our paper once more, which is contributing to the improvement of the manuscript. In the reviewed version of the paper, we will try our best to address the reviewer's specific technical comments by clarifying the assumptions underlying the model and their limitations, providing further justification for the treatment of sedimentation, habit dynamics, and water-vapour replenishment, and elaborating on the mathematical formulation and numerical implementation. We will also simplify the derivation of the mean-mass equation, remove unnecessary equations, and improve the description and consistency of the test cases and sensitivity analyses. Regarding comparisons with other models, we will seek to strengthen the existing evaluation where feasible, while distinguishing the objectives of the present theoretical study from the comprehensive inter-model comparisons that warrant a dedicated investigation.

Major Comment 1: Unbalanced inclusion of processes

We would like to clarify the scope of the present paper, which should primarily be viewed as a theoretical contribution in which we have introduced a Eulerian framework that directly resolves PDE fields for large-scale (in the sense of the number of plumes the model can resolve in a practical time scale) contrail simulations, with low computational time as its primary goal, while being able to handle more physical processes through low-order PDE fields. The additional contributions of the current paper are a dynamic habit parameterization through the correction of the ice-deposition formulas, emphasizing the importance of deposition kinetics for simulating ice supersaturation, and a reduced-order turbulent bulk-settling velocity formulation through the lens of multiphase flows. Notably, the paper also incorporates habit-dependent particle-level settling velocities, habit-dependent growth and sublimation, as well as a post-processing characteristic-based method for consideration of wind shear.

Major Comment 2. Monodisperse assumption in microphysical modelling

Response: We fully agree with the central physical point: ice crystals in contrails are not monodisperse, and size-dependent microphysical processes, particularly sedimentation, can produce differential crystal transport and gravitational size sorting. A monodisperse calculation cannot reproduce this spectral separation explicitly.

However, we would like to clarify an important distinction concerning the scope and mathematical structure of MultiCon. The present paper deliberately employs the monodisperse specialization of the framework as a first step for computational and theoretical demonstration. However, the underlying Eulerian formulation is derived from a population-balance equation (PBE) and can be extended to mass space to accommodate polydispersity directly. One of our ongoing research activities is to compare this explicit polydisperse extension with a reduced-order, proxy-polydisperse parameterization based on monodisperse prognostic quantities, building on the approach of Schumann et al. (2011) for accounting for particle-size-distribution effects on effective radius and extending it to other relevant microphysical processes. If the computational cost of the fully polydisperse formulation exceeds the threshold required to preserve MultiCon's substantial computational efficiency, the proxy-polydisperse formulation could provide a computationally efficient alternative for capturing important polydispersity effects without sacrificing the model's speed advantage.

Therefore, while it is true that the monodisperse assumption is a strong one for contrail-plume simulations, we do not believe that the current version of MultiCon is application-ready, not only because of polydispersity but also because of other effects, such as direct wind-shear feedback, ventilation, and hollowing morphologies, etc. The latter effects could even have an impact comparable to that of polydispersity under certain environmental conditions. We hope that this further clarifies the overall scope of the present work.

Major Comment 3. Neglect of wind shear

Response: We understand and appreciate the comment. In the version initially submitted to JECATS, we included the effect of wind shear using a characteristic-based method as a post-processing step, without coupling it to the model dynamics. This approach was adopted solely to enable a preliminary comparison with CoCiP. Following the reviewer's comment, we recognized that the following statement in the manuscript could be misleading: "Finally, because the separation ansatz employed in MultiCon prevents direct resolution of wind-shear effects, we account for wind shear by applying the method of characteristics (see Appendix H)." We will therefore revise this statement for clarity. In principle, wind shear should be coupled to the PDE dynamics, as suggested by the reviewer. One possible approach would be to perform a cross-sectional analysis of the plume, dynamically resolving the vertical dimension in MultiCon using the method of characteristics while prescribing the horizontal dimension using the formulation provided by Schumann (2012), which accounts for the dynamically evolving effects of wind shear. Nevertheless, as clarified earlier, the inclusion of wind shear in the current version is purely a geometrical tilting strategy implemented as a post-processing step to enable comparison with CoCiP. A more comprehensive treatment involving the dynamic coupling of wind shear to the governing PDEs is beyond the scope of the present work.

Major Comment 4. Sedimentation

Response: We appreciate the reviewer's comment. We agree that the equation requires verification. In this respect, after reviewing the available literature on this topic and the conditions under which ensemble settling velocities have been measured or simulated, we found that it may be possible to present a rough comparison with the following reference (Nielsen, 1993), which uses a random-walk algorithm. On the contrary, the references suggested by the reviewer do not seem to be fully compatible with our model for comparison purposes. We therefore intend to include the comparison with (Nielsen, 1993) in the revised version. We would also like to highlight that the bulk/ensemble settling velocity, containing loitering and preferential sweeping effects, is a complex phenomenon at the SGS level, requiring direct numerical simulations of multiphase-flow dynamics coupled with habit dynamics, mass-space (polydisperse resolution), growth/sublimation thermodynamics, etc. In this respect, the proposed reduced-order Burgers-equation model is viewed as a first step towards theoretical understanding of the ensemble/bulk settling behaviour. As shown in the manuscript, it can capture loitering and preferential sweeping behaviour, while the coupling with other microphysical processes has been deliberately encoded into its boundary conditions. We found this equation particularly interesting not only because of the features men-

tioned above, but also because it can yield a closed-form solution using the Hopf–Cole transformation.

Comment 5. Ice crystals habits

Response: As for the issue of habit dynamics, we would like to point out that the habit parameterization introduced in the paper incorporates nearly the state of the art in habit dynamics, although ice-habit dynamic modelling is a very complex subject and remains an ongoing field of research. We would also like to refer to a recently published work by **Dedekind et al. (2026, Atmospheric Chemistry and Physics)**, demonstrating the importance of deposition kinetics for simulating ice-surface supersaturation and contrail persistence using (almost) the same deposition-kinetics model as in MultiCon including extrapolation for the contrail temperature range. Please note that the core idea of habit evolution revolves around this deposition-kinetics model as it can be used to determine the directional surface supersaturation. However, **Dedekind et al. (2026, Atmospheric Chemistry and Physics)** have deployed the tangent hyperbolic deposition-kinetic model only to improve the classical deposition-kinetic of spherical particles. Interestingly, we take this a step further by adapting/coupling this deposition-kinetics model to habit geometry.

As for the relevance of habit dynamics to contrail studies, ice crystals are mostly non-spherical, and their habit can substantially affect depositional growth and sublimation, terminal settling velocity, and ultimately, contrail evolution and radiative forcing. Beyond the detailed justifications presented in the paper, and our previous response in *ACP Journal*, we note that even in contrail studies, models such as CoCiP already acknowledge different ice-crystal habits. However, due to modelling complexities, CoCiP considers habit classes only in the radiative forcing (RF) computation, assigning different habits based on ice-particle size.

Comment 6. Treatment of water vapour (appendix G)

Response: We thank the reviewer for this important comment. Since the earlier ACP version of MultiCon shares many features with the present JECATS preprint, we wonder whether some aspects of the comment refer to the earlier formulation.

An important difference is that, in the JECATS version, ice supersaturation, s_i, is coupled to the ice-microphysics equations through vapour depletion and particle growth, rather than being prescribed (see Eq. 12 of the paper). Nevertheless, we agree that the entrainment closure requires further clarification.

The physical motivation is consistent with CoCiP (Schumann, 2012), which accounts for ambient-water uptake through a bulk plume-mass budget as the plume expands, rather than by solving

a spatially resolved vapour-diffusion equation. Therefore, the mixing idea by itself in not new. Follwoing CoCiP, MultiCon adopts a reduced-order representation of ambient-water exchange which is now spatially resolved at least in z-dimension.

Physically, the effective entrainment timescale could also incorporate unresolved SGS turbulent mixing, potentially accounting for atmospheric stratification. However, the present formulation focuses on bulk entrainment associated with plume expansion and does not separately parameterize SGS mixing. Developing such a closure is beyond the scope of the current study and is left for the subsequent work.

Comment 7. Derivation of mean mass (appendix A4)

Response: We agree that the m-PDE seems to complicate the solution procedure and we will therefore remove it in the revised version for simplicity. However, please note that the m-PDE has been derived mathematically from the moment equations without any simplifications or additional assumptions. Therefore, it is a mathematically exact alternative to the first-moment PDE. Depending on the numerical approach, working with the m-PDE may offer advantages, as it can eliminate the need for a numerical safeguard against a zero denominator in subsequent analyses. However, there is a trade-off: the m-PDE itself requires a numerical safeguard against a zero denominator. Thus, this is a numerical choice rather than a theoretical or modeling choice.

Comment 8. Unusual behaviour in earlier simulations

Response: We would like to thank the reviewer for their observation. The spurious steady-steady arises from zero-denominator safe-guards in the code where column-integrated ice number and mass reach a threshold from which the code cannot process integration. This numerical artifact will be dealt with in the revised version.

Comment 9. Comparison with other contrail models

Response: We agree that a comparison with CoCiP alone was not sufficient. Following the suggestions of both reviewers, the revised manuscript compares MultiCon with CoCiP and with APCEMM, the two-dimensional model with sectional (polydisperse) microphysics and resolved wind shear used

by Akhtar Martínez et al. (2025). All three models are run from the same post-vortex state and in the same, fully specified atmosphere for a baseline and 27 one-at-a-time perturbations (Table R1, Figs. R1–R2 in the response to Reviewer #1). The results are consistent with the conclusion of Akhtar Martínez et al. (2025) that zero-dimensional models underpredict the lifetime optical depth. The time-integrated extinction of CoCiP is about one fifth of MultiCon's and APCEMM's is about twice MultiCon's. MultiCon therefore lies between the two, as the reviewer anticipated. Its plume structure and crystal sizes are close to APCEMM's and both models respond consistently to the perturbations. The remaining difference from APCEMM reflects processes that the present version does not resolve, in particular polydispersity and the horizontal inhomogeneity of the plume. The revised manuscript states this explicitly, so that the role of habit evolution and bulk settling is not over-interpreted relative to these limitations.

We ran the three models ourselves under identical conditions, instead of comparing with published curves, because the comparison is very sensitive to the initial state. With its own early-plume initialisation, APCEMM retains about 2 to 3 times more crystals and its integrated extinction roughly doubles, a larger effect than the differences between model formulations. For the same reason, and because of its computational cost, an LES comparison (e.g. Lewellen, 2014) is beyond the scope of this theoretical paper. In this revision, APCEMM is the higher-dimensional reference; the reasons are given in point (iii) of the general response to Reviewer #1. The new comparison will be presented in the results sections the revised manuscript.

Comment 10. Section on test cases and sensitivity studies

Response: Thank you for these detailed suggestions; Section 6 of the manuscript will be reorganised accordingly to run a sensitivity analysis (this analysis is not fully complete at time of writting this response). All test cases are now one-at-a-time perturbations of a single, fully specified baseline (Refer to Table R1 for details). The initial state is no longer prescribed but computed with the CoCiP wake-vortex parameterisation for a B737-800-like aircraft. We agree that LES results suggest larger initial cross-sections, so we added cases with $\sigma_{z,0}$ up to 120 m and $\sigma_{y,0}$ up to 150 m. Wind shear is now included, and so are the ice-supersaturated-layer depth, the humidity outside the layer, stratification, emission index and turbulent diffusivities. The figures now show the number concentration c_N instead of $g(z)$, all quantities carry units, and the convergence study specifies its physical set-up and includes Δt as well as Δz.

The sensitivity analysis will summarised with the extinction cross-section per metre, i.e. the contrail optical depth integrated across the plume, and its time integral.